Subject	Class	Paper	Reading materials on	Resource Person
Mathematics	D-I (H)	I	Theory of Equation	Dr. S. Ahmad Associate Prof.

Theory of Equation It is a branch of Mathematics which deals with the Solution of the Equation.

Factor theorem If α is a root of the equation $f(x)=0$, then the polynomial $f(x)$ is divisible by $x-\alpha$ [i.e. there will be no remainder]

Conversely if the polynomial $f(x)$ be divisible by $x-\alpha$, then α is a root of the equation $f(x)=0$

Proof. Given that α is a root of the equation $f(x)=0$

$$\therefore f(\alpha)=0 \quad \text{--- (1)}$$

Now divide $f(x)$ by $(x-\alpha)$ and let the quotient be denoted by Q and remainder by R.

$$\therefore f(x)=(x-\alpha)Q+R \quad \text{--- (2)}$$

Putting $x=\alpha$, we have

$$f(\alpha)=(\alpha-\alpha)\cdot Q+R=0+R=R$$

But from (1), we have $f(\alpha)=0$

$\therefore R=0$ i.e. there will be no remainder

and as such $f(x)$ is divisible by $(x-\alpha)$

Conversely let $f(x)$ be divisible by $x-\alpha$

$$\therefore f(x)=(x-\alpha)\cdot Q$$

Putting $x=\alpha$, we have $f(\alpha)=0$

$\Rightarrow$ shows that α is a root of the eq^n $f(x)=0$

Theorem: In an equation with real Coefficients, imaginary roots occur in Conjugate Pairs.

Proof. Let $\alpha+i\beta$ be a root of the eqn $f(x)=0$, in which the Coefficients are real.

Then $\alpha-i\beta$ must also be a root of the eqn $f(x)=0$

Since $\alpha+i\beta$ is a root of $f(x)=0$

$\therefore f(\alpha+i\beta)=0$ —— (1)

Let us divide $f(x)$ by $[(x-\alpha)^2+\beta^2]$ i.e. $[(x-\alpha)^2-i^2\beta^2]$ or by $(x-\alpha+i\beta)(x-\alpha-i\beta)$ and let the quotient be Q & remainder be $Rx+R'$

$$\therefore f(x) = (x-\alpha+i\beta)(x-\alpha-i\beta)Q + Rx+R' \quad \text{—— (2)}$$

Putting $x=\alpha+i\beta$

$$f(\alpha+i\beta) = [\cancel{\alpha}+i\beta-\cancel{\alpha}+i\beta][\cancel{\alpha}+\cancel{i\beta}-\cancel{\alpha}-\cancel{i\beta}]Q + R(\alpha+i\beta)+R'$$

$$\Rightarrow 0 = 2i\beta\cdot 0\,Q + (R\alpha+R') + iR\beta$$

$$\Rightarrow (R\alpha+R') + iR\beta = 0 \quad \text{—— (3)}$$

Equating real & imaginary parts in (3), we get

$$R\alpha+R'=0 \quad \text{—— (4)}$$

$$R\beta=0 \quad \text{—— (5)}$$

Now, β cannot be zero, for otherwise, the root will be real & hence from (5), we have $R=0$, therefore from (4), we have $R'=0$

Hence $f(x) = [(x-\alpha+i\beta)(x-\alpha-i\beta)]Q$

From above we conclude that $f(x)$ vanishes for $x=\alpha-i\beta$ and as such $\alpha-i\beta$ is also a root of the equation $f(x)=0$